Tests of stellar models

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Two tests are applied to the models for testing the pulsation calculations, etc. To test the equation of hydrostatic support I write it as

$$\frac{\mathrm{d}\ln p}{\mathrm{d}r} = -\frac{Gm}{pr^2}\rho \equiv f_p(x) \; , \tag{1}$$

I assume that the model is given on a mesh

$$0 = r^1 < r^2 < \dots r^n < \dots r^N = R_\mathrm{s} \; . \tag{2}$$

where R_s is the surface radius of the model. A similar notation is used for the variables (e.g., p^n) on the meshpoints. In general, the numerical scheme in at least `ASTEC`, and to some extent `ADIPLS`, is based on simple second-order schemes. Thus the basic numerical scheme is

$$\frac{\ln p^{n+1} - \ln p^n}{r^{n+1} - r^n} \simeq 0.5(f_p^n + f_p^{n+1}) \; . \tag{3}$$

A measure of how well this is satisfied is provided by

$$T(\ln p, r) = \frac{\ln p^{n+1} - \ln p^n}{r^{n+1} - r^n} \frac{2}{f_p^n + f_p^{n+1}} - 1 \; . \tag{4}$$

Note that, since the equations for stellar evolution are typically solved with m, $q = m/M$ or $\log q$ as independent variable, it is not guaranteed that $T(\ln p, r) = 0$ is satisfied, even though a difference scheme similar to Eq. (3) is used.

The second test concerns the buoyancy frequency as provided in the stellar model, or, equivalently, the density gradient. This is characterized by the Ledoux discriminant,

$$A \equiv \frac{1}{\Gamma_1}\frac{\mathrm{d}\ln p}{\mathrm{d}\ln r} - \frac{\mathrm{d}\ln \rho}{\mathrm{d}\ln r} \; . \tag{5}$$

In many codes, including ASTEC, this is computed from $\nabla - \nabla_\mathrm{ad}$, using thermodynamic quantities and, where needed, a numerical derivative of the hydrogen abundance; in that case the thermodynamic consistency of the equation of state is of course important. Alternatively a numerical derivative of ρ can be used. The issue is the consistency between A and the density ρ. Ian has proposed testing this based on $\mathrm{d}\ln\rho/\mathrm{d}\ln p$, obtained either from A or from the grid of p and ρ. Specifically, I define

$$D^{(A)}\rho^n = -\frac{A^n}{r^n f_p^n} + \frac{1}{\Gamma_1^n} \; , \tag{6}$$

and

$$D^{(\mathrm{IWR})}\rho^n = \frac{\ln\rho^{n+1} - \ln\rho^{n-1}}{\ln p^{n+1} - \ln p^{n-1}} \; . \tag{7}$$

Then the consistency check is (so far) defined with

$$T(\ln\rho, r) = \frac{D^{(A)}\rho - D^{(\mathrm{IWR})}\rho}{\max(D^{(\mathrm{IWR})}\rho, 0.1)} \tag{8}$$

(where the denominator takes into account the relatively rare cases of density inversions).